

# On MOOCs for Engineering Maths

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## Abstract

*There is a lot of buzz around the MOOCs initiative (Massive Open Online Courses). However, creating really useful resources for maths teaching and learning is a huge undertaking, and while filming lectures by good lecturers and performing computerised assessments is a good start, a lot remains to be done. The paper addresses some of the issues to be addressed: STEM students with weak backgrounds do not understand mathematical symbols and therefore are not able to follow internet lecturers who deliver material suitable for students with strong background. Automated tests have a value but only for assessing the so-called crystallised intelligence and to ascertain that students get education and not simply meaningless "degrees" they have to be encouraged to learn the material and not just train themselves to exams by studying previous exam papers. It is difficult to assure that students take the tests themselves. For these reasons, the next step is to develop multi-faceted resources, with different good quality teaching materials aimed at different audiences, supported by quality Cognitive Tutors (such as our e-PACT) to help students to practice the newly acquired skills "under expert supervision". These will never substitute a teacher but can certainly automate some of his/her most repetitious tasks.*

**Keywords:** MOOCs, STEM, math teaching.

## 1. Introduction

MOOCs stand for Massive Open Online Courses. These are *free online courses* offered by universities around the world (e.g., Stanford, Harvard, and MIT) to anyone who has access to internet. Several popular MOOC providers have emerged, such as Coursera, Udacity, EdX, and NovoEd that collaborate with universities to offer MOOCs on their platforms.

To date, ten large public university systems flagship universities have formed a partnership with Coursera: the State University of New York system, the Tennessee Board of Regents and the University of Tennessee systems, the University of Colorado system, the University of Houston system, the University of Kentucky, the University of Nebraska, the University of New Mexico, the University System of Georgia and West Virginia University. Some systems plan to blend online materials with faculty-led classroom sessions. Other leading online providers, too, have begun projects with public universities: edX, the non-profit collaboration founded by Harvard and the Massachusetts Institute of Technology, has teamed with the University of Texas and some California State University campuses, and Udacity, another Stanford spinoff, with San Jose State University. Other Universities plan to offer credit to students who take the courses online followed by a proctored exam on campus.

The enthusiasm for MOOCs is tempered by reservations. Some faculty resistance has emerged recently against using online materials, even if they are blended with classroom work. Recently, 58 Harvard professors wrote a letter seeking the creation of a new committee to consider the ethical issues related to edX and its impact on higher education. In this paper we discuss pedagogical issues and opportunities associated with using MOOCs to deliver engineering maths, particularly to students from disadvantaged backgrounds.

## 1. General pedagogical issues associated with MOOCs

### 1.1. Educating disadvantaged students

In 2012 Gary S. May, Dean of the College of Engineering at the Georgia Institute of Technology wrote: "MOOCs offer a huge opportunity to investigate how to use technology to more effectively educate students. They could potentially serve as laboratories to conduct experiments that might reinvent education. How can student learning be optimized in an online environment, and what is the best role of the faculty member in such an environment? Is the "flipped

classroom" – i.e., using online lectures as preparation work for in-person interactions at multiple locations – a viable approach?" [1] Would it not be great to have every student introduced to the subject content by a top professor? Fueled by this thought, Coursera initially recruited as partners only the Ivy League members in the Association of American Universities. Houston Davis, the chief academic officer of the University System of Georgia, said that while the system would start with just a handful of Coursera courses next fall, he hoped a full menu of general education courses — the gateway classes usually taken in a student's first two years — would eventually be available online through Coursera, for sharing by all the campuses [2]. Moreover, the Stanford Professor Thrun, one of the creators of Udacity suggests that "in 50 years there will be only 10 institutions in the world delivering higher education" [3]. These unrealistic initial claims would be acceptable if left to academics and journalists to discuss. Unfortunately, they supported the fallacy popular with far too many HE managers who find it expedient to claim that nobody needs local professors to teach undergraduates, this can be done on the cheap –using teacher assistants or on-line courses. However, many academics would argue that it is precisely in the first two years of their academic studies that students – and particularly disadvantaged STEM students – need intensive guided teaching. This can be delivered only in person and only by the most experienced teachers capable of not just "talking" – delivering content, but mainly of "listening" – and readjusting their delivery depending on the student immediate feedback/

"Our first year, we were enamoured with the possibilities of scale in MOOCs," said Daphne Koller, one of the two Stanford computer science professors who founded Coursera. "Now we are thinking about how to use the materials on campus to move along the completion agenda and other challenges facing the largest public university systems." The company is eager to work with a broader range of institutions, to see how its materials can help more students complete their degrees [4]. Despite Professor Thrun's predictions, while some Universities still intend to use existing Coursera materials developed by faculties at elite universities, others begun to say that they would expect that their own faculties will develop materials for the Coursera platform, making them available at campuses system wide and beyond. Faculty members will be able to customize existing courses, adding their own lessons and refinements, the company said. What led to this change of tack? The first sets of MOOCs data have shown that the initial expectations were unrealistic. The University of Edinburgh has reported in 2013 that "Thirty-three per cent of respondents (a subset of their MOOC users - LF) were between 25-35 years old and were mostly in the "teaching and education" field or students at university. 70% reported having completed an academic degree, a larger percentage than organisers expected." [5] Similar reports have been produced by Harvard and MIT: "Average course certification rates are ...6% among all registrants in the course (841,687 registrants with 597,692 unique users - LF)...The most typical course registrant is a male with a bachelor's degree who is 26 or older ... (31%)...33% report a high school education or lower; 6.3% report that they are 50 or older; and 2.7% have IP or mailing addresses from countries on the United Nations list of Least Developed Countries." It is not clear whether any certificates of attainment have been secured by those who had no prior degree.

Of course, the low success rates do not diminish the value of MOOCs, not for the learners genuinely interested in learning, having reached the pre-requisite level and not able to attend the classes in person. However, for the first time academics at the top institutions – just like those who work in the institutions of widening participation had to do for a long time - have to draw managers' attention to the fact that educating the disadvantaged is much harder than educating students with strong prior backgrounds and for the first time they have to start questioning the pedagogy required to deliver higher education to those who received no good education prior to registering o their course. The questions are particularly pertinent when addressing maths education of STEM students. It is well known that a significant number of STEM freshmen do not understand mathematical symbols. Even the equality sign is a problem and few know the difference between an expression and equation. The Sheffield Hallam University has recently reorganised their whole calculus curriculum, devoting each lesson to a particular mathematical symbol and treating the standard calculus concepts as examples of mathematical symbols' use, coupled with the well-known phenomenon of maths aversion is it realistic to expect that this large group of students could follow Stanford, MIT and Harvard professors who deliver material suitable for students with strong mathematical background and use traditional lecture style? Only proponents of the discovery learning could think that the answer could be positive, but this mysteriously popular educational approach has been repeatedly shown to be ineffective with novice learners [6]. Anyone who taught maths to such learners would give a negative answer even before they saw any MOOCs data – simply because they know how many hurdles have to be overcome before they turn into competent engineering students: they have to be guided towards understanding the mathematical language, connections have to be established between the neuropaths that would allow them to use this newly acquired language, engage in logical reasoning and critical thinking and be introduced to schemas for integrating new information with their prior knowledge. As I argued in several previous publications [7] - [11] none of this can be achieved without a dedicated teacher giving [12] and receiving (sic! – see [13]) a constant feedback. Ideally, his or her findings have to be correlated with those of the rest of the faculty teaching the same students The failure to provide strong learning support can actually result in the loss of learning [6].

## 1.2 Assessing the state of student knowledge

Assessment modes are widely discussed in the context of MOOCs. Formative assessment represents the greatest challenge, with humanities teachers advocating a greater emphasis on ongoing self, peer and even crowd grading. Even computer science courses are planned to have an element of peer review [14]. For the summative assessment, it is proposed that the final exams be proctored and that the identity of the test taker be authenticated, whether in-person at a testing center or using a webcam proctoring service. Automated tests have a value, mainly for assessing the so-called crystallised intelligence and to ascertain that students get education and not simply meaningless "degrees" they have to be encouraged to learn the material and not just train themselves to tests and exams by studying previous test and exam papers. In my practice that requires a lot of personal interaction and it is not clear yet whether those running massive on-line courses can learn to change their students' attitude to learning.

Finally, right now, cheating is much more difficult to prevent in the online world than in a physical classroom. In a recent panel discussion on online education, Dave Patterson, who taught a MOOC at the University of California at Berkeley, described technological evidence that indicates such cheating is "unbound." Purveyors of MOOCs will have to develop sophisticated tools and processes to prevent students copying from one another or having somebody else to sit their tests for them [1].

## 2. Cognitive maths tutors

As mentioned above, MOOCs introduce the pedagogical dimension into the HE discourse. While "math wars" have led to many changes in the US school curriculum, most University teachers engaged in maths teaching are barely aware of current pedagogical thinking and findings, and in my experience, the managers – if engaging in pedagogical matters at all - are just regurgitating some half-digested constructivist arguments and popular myths [15], usually exhibiting a lack of comprehension of the issues involved. Apart from offering better structured courses, supplemented with advanced simulation tools that aid learning, MOOCs can open a door to advanced cognitive tutors, introducing University maths teachers to more efficient ways of teaching students from disadvantaged backgrounds.

Three major paradigms are used by designers of current mathematics tutors, Computer-aided assessment (CAA), Computer-aided instruction (CAI) and Intelligent computer-aided instruction (ICAI). The CAI type tutors still represent an overwhelming majority of hundreds of mathematics software packages that can be found or found advertised on the web. A good example of CAA is *maths e.g.* [16], which produces a large supply of questions generated at runtime, each with very complete feedback, including a fully-worked solution if a student gets an answer wrong. Examples of CAI applications include guided drill and practice exercises, computer visualization and computer-facilitated communication between students and teachers. Well known current CAI type mathematics tutors for engineering students are HELM [17] and MyMathLab [18]. HELM, freely available online, offers digitized lecture notes enhanced with hypertext, worksheets and multiple choice tests, MyMathLab is a commercial product, relying on digitized lecture notes, providing instruction in a variety of media, practice as well as tutorials, assessment and progress monitoring. MyMathLab's homework and practice exercises are correlated to the exercises in the textbook, and they regenerate algorithmically to give students unlimited opportunity for practice and mastery. Exercises include guided solutions, sample problems, and learning aids for extra help at point-of-use, and they offer helpful feedback when students enter incorrect answers.

Open object oriented learning environments are a newer development in CAI that is meant to provide users with one platform which allows an easy access to various graphical, modeling and pedagogical tools (agents) and an easy interaction between different learners as well as learners and human tutors. While an exciting challenge to computer science and potentially an interesting tool to use in a classroom, they are subject to the same criticisms as offered above on old style CAI.

The ICIA tutors come closer to implementing constructivist epistemology and in particular, guided teaching. The architectures of classical intelligent or Cognitive tutors include procedural representations, conceptual structures and production rules while newer architectures also have multiple soft constraints (e.g. neural networks, fuzzy production systems) as well as dialog moves generators. Propositional representations, neural networks and fuzzy production systems are relevant only to tutors involved in processing natural language. Procedural representations are used when the ordering of reasoning steps is important, as it is when teaching mathematics. Production rules are relevant to all cognitive tutors, since according to ACT-R [19] theories, cognitive skills are based on production rules. These two representations have been thoroughly discussed by designers of two major Cognitive Tutors developed to instruct in

technical subjects, AutoTutor [20] and the Carnegie Learning System [21]. They engage freshmen in a Socratic dialogue based on the natural language by simulating the dialogue moves of human tutors. They indicate to students whether their answer is correct, can generate hints, divide the problem into different steps and provide proper feedback for each. The current versions are designed to help college students learn topics in physics, computer literacy and algebra.

Both groups emphasize that the main difficulty lies in designing a Cognitive Model, that is, the part of the tutor that is charged with the task of interpreting the student performance and making instructional decisions based on this interpretation. To clarify the concept of a Cognitive Model, the one used in AutoTutor is based on the idea of curriculum scripts. These are “well-defined, loosely structured lesson plans that include important concepts, questions, cases, and problems” to be covered in a particular lesson. For example, the curriculum script for AutoTutor on computer literacy currently covers three macrotopics, hardware, the operating system, and the Internet. There are 12 topics within each of the 3 macrotopics (36 in total). The script includes *36 computer literacy questions and/or problems and 36 topic related questions/problems*. It also includes *36 Ideal Answers* that correspond to each of the 36 topics. The *quality of any given learner contribution is determined by matching the learner contribution to each aspect* and all possible combinations of aspects in a particular Ideal Answer. Additional information contained in the curriculum scripts includes: (1) *anticipated bad answers for each of the 36 topics*, (2) *corrective splices (i.e., correct answers) for each anticipated bad answer*, and (3) numerous dialogue moves (i.e., elaborations, hints, prompts, prompt responses and summaries) that are related to the aspects in the Ideal Answers. All content in the curriculum scripts is written in English, as opposed to computer code. Therefore, a teacher or other individual who is not an expert programmer can easily author a curriculum script.

While holding an exciting promise, the current ICAI technologies, even as advanced as AutoTutor and the Carnegie Learning System, are still immature when it comes to teaching and learning, both because their repertoire is very limited and because the dialogue often leaves a lot to be desired. The author’s group made its own attempt to design and develop a demonstrator of an electronic Personal Algebra and Calculus Tutor (e-PACT).

## **2.1 Architecture of e-PACT**

The requirements employed in design of e-PACT are similar to the requirements for the Carnegie Learning System:

### **Simple, Straightforward GUI**

- Straightforward presentation of mathematical symbols
- No distracting images or photos, minimalist presentation
- Intuitive
- Easy to use

### **Just-in-time Feedback**

- Hints are contextual and oriented towards helping the student to follow key steps in the problem.
- Immediate feedback enables the student to self-correct and leads to more effective learning and applying of the mathematics
- e-PACT recognizes the most common student errors and responds appropriately.

### **Achievement monitor**

- As a student becomes more proficient in a skill, e-PACT progresses him/her to a higher level of difficulty.
- Teachers can view off-line an immediate snapshot of each student’s progress and full dialogues.
- Students receive a dynamic, strong motivator to succeed.

Let us describe the architecture designed to meet these requirements. The natural language processing that hampers the dialog in such systems as AutoTutor is not addressed by e-PACT and therefore, it utilizes only the following architectural features: Cognitive Model, Procedural representation (based on Decision Trees), Object-Oriented Design, Production Rules (if – then or condition-action pairs) and Dialog Moves.

### 2.1.1 Cognitive Model and Procedural Representation

e-PACT is meant to utilize the Socratic lecture/tutorial model of teaching and make use of a scaffolding tool not usually adopted in mathematics instructions, a Decision Tree. The Lectures and Summaries of lectures are to be included under Help as texts containing didactic descriptions supported by examples, but some conversational aspects of the Socratic methodology built around the author's experience of how students learn and think are to be automated. To widen the Tutor repertoire the idea of a database/back-end is to be abandoned: e-PACT is meant to generate a large number of problems at random and programmed to "discuss" any of them. Thus, on the one hand, the e-PACT's cognitive model is simpler than any of those used in AutoTutor or CLS but on the other hand, unlike them, it generates and discusses a large number of possible exercises (of prescribed types). This can run into hundreds.

e-PACT is designed to contain Intelligent Context Aware Parsers that recognize common errors and misconceptions and dynamic Decision Trees that sequence solution steps and guide the student through them with prompts and comments. e-PACT is to tailor its hints and responses to specific mistakes in student answers. Whatever student's turn, e-PACT is to present relevant and effective comments to build his/her mastery of the subject. By constructing human-like dialogs and using correct verbalization of mathematical processes e-PACT aims to develop – among other things - students communication skills.

Whether commenting on a particular answer or engaging student in a Decision Tree based dialogue, it is difficult for e-PACT creators to ascertain that an intelligent feedback is provided in all cases, and that this is always done in an understandable and conversational manner. The problem is compounded by the fact that e-PACT is meant to interpret many different styles of mathematical input, allowing for many different conventions and a lot of sloppiness. This is a conscious choice, since "in tutorial systems, effective progress in teaching the problem-solving target is frequently hindered by expressive sloppiness and low-level errors made by the student, especially in conventionalized expressions such as formulas." [22]. Thus, the current e-PACT prototype is tolerant to a number of spaces used by the user, it can interpret a function whether the argument is bracketed or not (such inputs as  $\sin(x)$ ,  $\sin x$  and  $\sin x$  would be treated the same); if an expression is bracketed more than once, it just generates a gentle warning that the input contains extra brackets; and if a bracket is missing here or there, this is also handled through warnings rather than error messages. If an error is of the type expected of a dyslexic student, say the input is  $e^x$  rather than  $e^x$  the prototype sends a detailed message on the corresponding mathematical convention, drawing the student's attention to the fact that in mathematics the order and position of symbols is often imbued with a particular meaning. The e-PACT prototype already "knows" enough algebra to be able to comment on such input as  $p^{-1}$  whether it is given in that form or as  $\frac{1}{p}$  and in its messages will use the language appropriate to the form chosen by student. While this tolerance provides for better usability it makes interpreting the student performance and arriving at instructional decisions based on this interpretation a challenging task.

### 2.1.2 Object Oriented Design

Taking into account the specific area of expertise, the e-PACT' object oriented design is based on the classes that model mathematical objects. For example, in the current prototype the class Function Elementary contains the same elements as elementary functions used in undergraduate mathematics, *function name*, *argument* and *power index* (if applicable); the class Sum of Two Functions models *addition of functions etc.* The e-PACT's architecture as engineered at the highest level is shown in Figure 1a): the Random Problem Generators are meant to generate a variety of exercises, Solvers are meant to solve them rather than have their solutions stored in a database, Intelligent Input Readers are meant to interpret a large class of possible inputs, including the ones containing extraneous symbols and Solution Comparators are meant to compare the expected answer with the one provided by a student. An e-PCT Controller is meant to manage the GUI interaction between the user and Core, including interaction with the dynamic Decision Trees to be affected via a Decision Tree Based Dialogue module (see Figure 1 b)). It is planned that in the full version of e-PACT the Controller takes into account the particular difficulties experienced by the user and adapts by taking him/her to the topics that require extra revision.

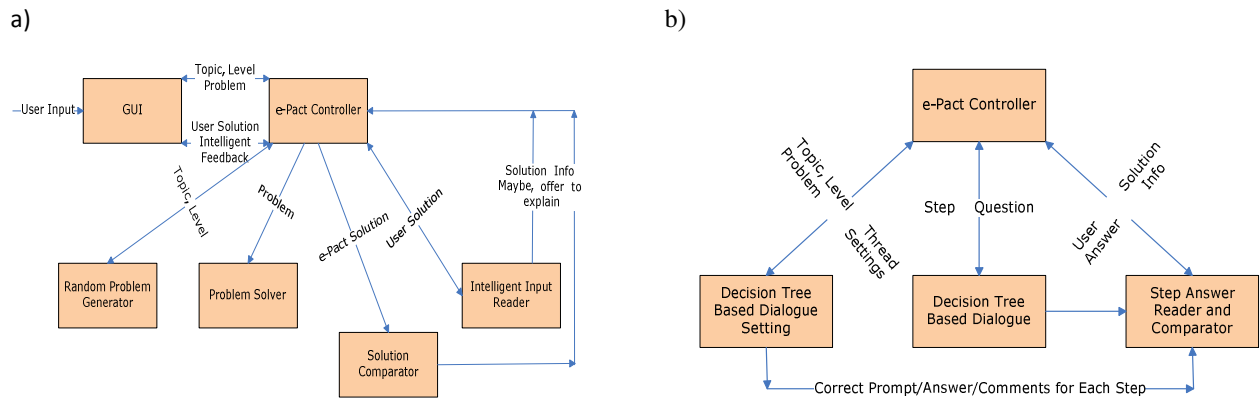


Figure. 1.a) e-PACT Architecture: assessment of student's final solution. b) Architecture of the Decision Tree Based Dialogue.

### 2.1.3 Production Rules

Comparators implemented in e-PACT are not meant to involve any statistical analysis but compare identifiable parts (aspects) of the mathematical objects, such as *sign*, *coefficient*, *function name*, *argument*, *term factor*, *sum*, *product* etc. All instructional decisions and messages are meant to be based on this comparison and therefore, to be context-dependent.

For this reason, e-PACT messages are meant to utilize information from different pieces generated by the Compare methods in different classes, such as Elementary Function, Simple Function, Sum of Two Functions, Product of Two Functions etc., each responsible for its own portion of the mathematical input. The messages are meant to be formatted to alleviate comprehension, using indents and helpful connectives, depending on the number of identifiable aspects in the student answer and the number of mistakes in each.

When creating a Cognitive Tutor, the advantages of using the object-oriented design rather than a database are two-fold: we can deal with relatively large classes of problems at once and extra levels of difficulty and extra complexity can be added without affecting the design functionality. This implies that the Cognitive Tutors designed in this manner are easily maintained and enhanced. The clarity of design is also an advantage: the Core classes model traditional mathematical concepts and thus new researchers can be easily integrated into the team.

### 2.1.4 Dialogue moves

The types of dialogue moves used in e-PACT are similar to the ones used in other Cognitive Tutors but since responses are only half pre-programmed their implementation is more involved. The types of moves used in the current prototype are positive feedback, negative feedback, splice, elaboration, explanation, summary, prompt, guess and connection. A sample screen shot is presented in Figure 2.

## 3. Conclusion

It has been argued that at present only a minority of academics utilize modern pedagogical principles when teaching engineering maths. While there is no need to concentrate on these principles when educating students well prepared for college work the issue becomes important with the students from disadvantaged backgrounds. The first MOOCs have been developed by professors from elite Universities who have not had to think about these issues in the past, but in principle MOOCs platforms could be used to provide courses more suitable for those who need remedial teaching. In particular, teaching of mathematical concepts necessary for succeeding in engineering maths and therefore many engineering courses could be supported by formative assessment that could be provided by cognitive tutors, such as e-PACT, that has been discussed in some detail.

**Tutor**  
FILE LEVEL REQUEST MATH SYMBOLS HELP

DIALOGUE WINDOW

STUDENT: Please generate an exercise.

TUTOR: Find  $\frac{d}{d\ln(u)} \cos(\ln(u))$ .

STUDENT: help

TUTOR: Well, let us solve the problem step by step.

STUDENT: Please explain.

TUTOR: To understand the solution we use the DECISION TREE.  
Start from the top. Your answers will suggest which arrows to follow  
What is the first question you have to ask yourself?

STUDENT: Q1

TUTOR: OK, the question to ask is  
"What is the differentiation variable and what is the function to differentiate?"  
Please specify the differentiation variable.  
Type the answer and press SUBMIT button.

STUDENT: ln u

TUTOR: This answer is correct!  
Please specify the function you are asked to differentiate.  
Type the answer and press SUBMIT button.

STUDENT: -sin ln u

TUTOR: It seems that you have tried to type the solution to the problem. Am I correct?

STUDENT: y

TUTOR: You must learn to answer the question you have been asked and not jump ahead.  
Nevertheless, I will accept this attempt.  
Your answer is correct!  
Please move the cursor to the MENU BAR. Click first on the REQUEST option and then on the NEW EXERCISE.

PROBLEM AND SOLUTION WINDOW

PROBLEM: Find  $\frac{d}{d\ln(u)} \cos(\ln(u))$ .

SOLUTION:  $\frac{d}{d\ln(u)} \cos(\ln(u)) = -\sin(\ln(u))$

Summary

DECISION TREE

```

graph TD
    Start([df(x)/dx = ?]) --> Q1([Q1: What is the differentiation variable? Look after d at the bottom.  
What is the function to differentiate? Look after d at the top.])
    Q1 --> Q2([Q2: Is f(x) a constant or another table function  
of the differentiation variable?])
    Q2 -- Yes --> Table[The answer is in the table]
    Q2 -- No --> Q3([Q3: What is the last operation?])
    Q3 -- +, -, / --> Q4([Q4: Is one of the factor const ≠ 1?])
    Q4 -- Yes --> Separate[Separate terms]
    Q4 -- No --> Product[Product rule or quotient rule]
    Q3 -- * --> Q5([Q5: Is f(x) composite?])
    Q5 -- Yes --> Chain[Chain rule]
    Q5 -- No --> Composite[Make it composite]
  
```

Figure 2. A screen shot of a recent Differentiation Tutor with a sample dialogue on Level 2

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