The new scheme of a formalization of an expert system in Teaching

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Abstract

Educational technologies are now one of the most important part of the learning process. The present paper refers to the description of expert systems in the education in particular in the teaching mathematics. The new scheme of a formalization of an abstract system based on tools of Algebraic System's Theory, Group theory and System Approach is constructed and it is shown how to use it in teaching. More particularly the scheme of an expert system for testing pupils in mathematics based on the tools of Group Theory is constructed. This scheme contains an algorithm of compilation errors database which is described by the language of the first order of Group Theory in the conjunction that a composition of mistakes is an associative binary operation. Theorems of true actions' description in the process of solving tasks and of errors' description in the process of solving tasks are proved. Using proposed in the article approach to the formalization of the General theory of systems with the help of the theory of algebraic systems one can obtain substantial results in various fields of activity, not only in the teaching but in the modeling of economic phenomenon and processes and in the modeling of processers of the different nature as applications.

Keywords: *algebra, algorithm, educational technology, modeling.*

1. Introduction

First of all it is important to choose the basic concept by the help of which the solution of the stated problem can be realized. In our paper this is a new approach to a formalization of abstract systems is developed.

Let's concentrate short on the history of the studied problem.

Early known results in the formalization of system approach's field are considered in [1]. M. Mesarovich and Y. Takahara formalize a system in terms of its properties or otherwise in terms of the relationships between the properties of the system. Their approach used binary relations. This is well consistent with the essence of the system research aimed at clarifying the organization's systems and relationships within it. However, the binary relations on abstract sets is not studied well enough to receive substantial theorems of General Systems Theory.

In 1970 A.I. Maltsev's famous book "Algebraic Systems" was published over which he worked for about 20 years. [2] Also we note that A.G. Kurosh (Conclusion to the first edition) [2], wrote that it is impossible to limit the use of the theory of groups (a group is a kind of an algebraic system) by questions of it's nearest areas. So it seems to us that the formalization of the system approach in the terms of Maltsev's algebraic system's theory is able to capture the essence of the General System's theory and to get deep results in it and in many different areas of science.

In 1992 – 1993 on the study of the works of academician Y.L. Ershov there was proposed the method of allocation and study of pure (clean) embedding in a special class of algebraic system that is in groups. [4]

That method has allowed to transfer and generalize well-known results of the purity's theory of Abelian groups to the case of arbitrary non-Abelian groups.

In 2000-ies the scheme of a formalization of an abstract system in terms of group theory has been applied to closed economic systems to forecast the number of final states of a system. [5]

In the present paper pure (or clean) embedding within the meaning of this notion not distort domestic system relations are used to distinguish relations in systems according to their nature, i.e. to classify links in abstract systems.

2. The new scheme of a formalization of a system approach

We will be mostly interested in the question of changes in the properties and the state of the system under the influence of the interaction of the factors affecting it, therefore, we propose the following definition of the algebra of factors of a system.

Definition 1. Under the algebra of factors a system will be understood algebra $\bar{A} = \langle A | \{f_\alpha^{n_\alpha} | \alpha \in \Gamma\} \rangle$ with the fundamental set of factors A and the set of operations $\{f_\alpha^{n_\alpha} | \alpha \in \Gamma\}$ describing the interaction of the factors.

Definition 2. Subalgebra $\bar{B} = \langle B | \{f_\alpha^{n_\alpha} | \alpha \in \Gamma\} \rangle$ of an algebra $\bar{A} = \langle A | \{f_\alpha^{n_\alpha} | \alpha \in \Gamma\} \rangle$ is called P - pure (P - clean) in $\bar{A}$ if every homomorphism $\bar{B} \xrightarrow{\varphi} \bar{C}$ of the subalgebra $\bar{B}$ into $\bar{C}$, where $\bar{C}$ is an algebra of the signature $\{f_\alpha^{n_\alpha} | \alpha \in \Gamma\}$ of the of $\bar{A}$ and $P(\bar{C})$ is true and P is a predicate on the class of algebras of the signature $\{f_\alpha^{n_\alpha} | \alpha \in \Gamma\}$ closed under taking subalgebras and factoralgebras, can be continued to a homomorphism $\bar{A} = \langle A | \{f_\alpha^{n_\alpha} | \alpha \in \Gamma\} \rangle$ into $\bar{C} = \langle C | \{f_\alpha^{n_\alpha} | \alpha \in \Gamma\} \rangle$ i.e. the following diagram is commutative:

$$0 \to \bar{B} = \langle B | \{f_\alpha^{n_\alpha} | \alpha \in \Gamma\} \rangle \xrightarrow{\varphi} \bar{A} = \langle A | \{f_\alpha^{n_\alpha} | \alpha \in \Gamma\} \rangle \qquad (1)$$

$$\bar{B} \xrightarrow{\alpha} \bar{C} = \langle C | \{f_\alpha^{n_\alpha} | \alpha \in \Gamma\} \rangle, \qquad \bar{A} \xrightarrow{\beta} \bar{C}$$

that is $\beta\varphi = \alpha$

Let's concern predicates' classification.

The predicate *P* highlights the statistical properties of the system if it does not depended on the time or on changes of other external factors of the system. For example purity in the class of abelian groups or in the class of all groups or in the class of all algebras of a fixed signature is static. The predicate *P* can allocate dynamic system's property if it depends on the time or on changes of other external factors to the system. For example, while considering the financial system, the predicates providing financial stability, the legal sector of the economy, etc., are dynamic ones. A predicate, unlike numerical indicators, allows to characterize the investigated properties in a single holistic complex as numerical indicators, and synchronized relationships, in the dynamics, if it is a dynamic predicate, and in statics, if it's a static predicate.

Let's concern the dynamic predicate's formalization.

Definition 3. The predicate *P* is a dynamic one if it is represented in the form $P(A,t)$ where *t* is time or another external to the system changing factor. The continuous and discrete scale can be defined for *t*.

Definition 4. Subalgebra $\bar{B} = \langle B | \{ f_\alpha^{n_\alpha} | \alpha \in \Gamma \} \rangle$ of an algebra $\bar{A} = \langle A | \{ f_\alpha^{n_\alpha} | \alpha \in \Gamma \} \rangle$ is called P - pure (P - clean) in $\bar{A}$ if every homomorphism $\bar{B} \xrightarrow{\varphi} \bar{C}$ of the subalgebra $\bar{B}$ into $\bar{C}$, where $\bar{C}$ is an algebra of the signature $\{ f_\alpha^{n_\alpha} | \alpha \in \Gamma \}$ of the of $\bar{A}$ and $P(\bar{C}, t))$ is true and $P(t)$ is a predicate on the class of algebras of the signature $\{ f_\alpha^{n_\alpha} | \alpha \in \Gamma \}$ closed under taking subalgebras and factoralgebras, can be continued to a homomorphism $\bar{A} = \langle A | \{ f_\alpha^{n_\alpha} | \alpha \in \Gamma \} \rangle$ into $\bar{C} = \langle C | \{ f_\alpha^{n_\alpha} | \alpha \in \Gamma \} \rangle$ i.e. the following diagram is commutative:

$$0 \to \bar{B} = \langle B | \{ f_\alpha^{n_\alpha} | \alpha \in \Gamma \} \rangle \xrightarrow{\varphi} \bar{A} = \langle A | \{ f_\alpha^{n_\alpha} | \alpha \in \Gamma \} \rangle$$

$$\alpha \searrow \qquad \beta \qquad \swarrow \qquad (2)$$

$$\bar{C} = \langle C | \{ f_\alpha^{n_\alpha} | \alpha \in \Gamma \} \rangle$$

that is $\beta\varphi = \alpha$ and predicate P satisfies the following conditions:

1. $P(A,t), A_1 \le A \Rightarrow P(A_1, t)$

2. $P(A,t), t_1 \le t \Rightarrow P(A, t_1)$

Let's concern the determination of the point of an equilibrium of the system. The proposed approach allows us to determine the equilibrium point of the system, as well as stable and not sustainable systems.

Now consider a system with full implementation of all links that satisfy the predicate P. An algebraic system of the signature *Ω* which is injective with regard to all *P*-pure sequences in the class of all algebraic systems of the signature *Ω* is a system with full realization of all links that satisfy the predicate *P*.

Let's concern inverse limits of systems and the embedding of a system into a system with a full implementation of the R relations. Generalizations of theorems on the structure of algebraically compact groups and the theorem that any reduced abelian group can be embedded as a pure subgroup into an algebraically compact abelian group (inverse limit of cyclic groups) are effective to build the *P*- pure embedding of a system into a system with full realization of the *P* - links. In addition, for General Systems Theory, this approach provides an opportunity to prove the analogue of the theorem that an algebraically compact abelian group is allocated as a direct summand of the group containing it as a pure subgroup: *P* – pure subsystem with a full implementation of the P - links in it is a retract of a system containing it. So we obtain the following theorem.

Theorem. Any system with a full realization of the *P* - links can be offline.

3. Special case: the factors affecting the system, define the group.

In this case, the algebra $\bar{A} = \langle A | \{ f_{\alpha}^{n_{\alpha}} | \alpha \in \Gamma \} \rangle$ with the main set of factors A, and the set of operations $\{ f_{\alpha}^{n_{\alpha}} | \alpha \in \Gamma \}$ describing the interaction of the factors is the group $\bar{A} = \langle A | \circ, \ ^{-1}, e \rangle$, where $\circ$- is the composition of the factors, i.e. the consistent implementation (realization) of factors, $^{-1}$is the operation of taking (implementation) reverse factor, *e* is a neutral factor.

Let's consider the meaning of purities and examples of P – purities in the class of all groups. Chart (1) has the following meaning in the class of groups: epimorphic images of B and A in the class of all finite groups are one and the same. For *P* –purities the meaning runs as follows: *B* and *A* have the same epimorphic images in the class of all groups satisfying the condition *P*.

Examples:
- *P* allocates the class of all finite groups in the class of Abelian groups, get the usual purity in the class of all Abelian groups;
- *P* allocates the class of all Abelian groups in the class of all groups;
- *P* allocates the class of all finite groups in the class of all groups;
- *P* highlights the diversity in the class of all groups i.e. the class of groups closed under subgroups, homomorphic images and Cartesian products, such as Burnside's variety of all groups of the exponent (indicator) *n* defined by the identity$x^n = 1$, the variety of nilpotent groups of class of nilpotent is not more than *n*, soluble groups of length not exceeding the number *l*, etc.

Let's concern the automorphism group of factors determining the system. Let $\bar{G} = \langle G | \circ, \ ^{-1}, e \rangle$ be the group of all factors which describe the system *Ğ*. Then the automorphism group $Aut(\bar{G})$ shows all of the possible links structure factors acting on the system *Ğ* just the same as $\bar{G}$.

4. The new scheme of an Expert System for testing pupils in mathematics based on tools of Group Theory

This formalization allows us to construct the following scheme of an Expert System for testing pupils in mathematics based on tools of Group Theory. It runs as follows.

Algorithm of compilation errors database runs as follows. Algorithm of compilation errors database is described by the language of the first order with the signature $\Omega = \langle *, ^{-1}, e \ \rangle$ of Group Theory in the conjunction that a composition of mistakes is an associative binary operation.

Table of symbols: *n* –the number of tasks, $Q_1, Q_2, \dots, Q_n$ – all tasks of database. Expert writes down all atomic errors each of which contains only one error action $\{ m_j^i | i = 1, \dots n, j = 1,,, q_i \}$ where i is the number of a task, j is the number of an error for a given task. A composition that is the consistent implementation of error actions, is an error: $m_j^i * m_r^s, i, s = 1, \dots n, j, r = 1,,, q_i$. If an atomic error m_j^i is made then the reverse action is an error which is designated by $(m_j^i)^{-1}$ or by $m_j^{i^{-1}}$ and is named a reverse error. Evidently $((m_j^i)^{-1})^{-1} = e$, where e is neutral error or neutral element. Pupil's answer on a test may be nominally true but it can contain neutral error, for example, $m_j^i * m_j^{i^{-1}} = e$. Such answer could not be considered as the right decision. Thus the full system of errors of a given task can be described by a group) $\mathfrak{G} = \langle G,*,^{-1}, e \rangle$. Then $\mathfrak{G}$ is a free group of finite rank $k = n\sum_{i=1}^{n} q_i$ with $\{m_j^i | i = 1, \dots n, j = 1,,, q_i\}$ as generates.

The following theorem of errors' description is proved.

Theorem of errors' description. The set of all errors is defined by no more than two combinations of words which are finite length's compositions of atomic errors.

Now consider Search and analyses of mistakes and typification of errors and scaling. Experts can single out standard errors which can be written down as composition of atomic errors that is as words $\omega_\propto(x_1, \dots, x_n), \propto \in \Gamma$, in alphabet $\{m_j^i | i = 1, \dots n, j = 1,,, q_i\}$ thus setting a group $\mathfrak{G} = \langle G \ | \omega_\propto(x_1, \dots, x_n) = e, \propto \in \Gamma \rangle$ with defining relationships – the group of errors of a task. Van Kampen's lemma allowed to see the group of errors of a task and to analyze it. Besides this one can build the subgroup lattice of this group giving the opportunity to represent graphically all errors in task number *i* solution.

Introduction of atomic errors allows to scale the area of errors by their length without taking into account neutral errors that is without species compositions $a \ * a^{-1}$.

Algorithm of compilation of knowledge base and search and analyses of true algorithms of decisions run as follows.

For every task expert forms the full set of true actions by the following algorithm. Experts can single out all atomic true actions $T = \{a_j^i | i = 1, \dots n, j = 1,,, l_i\}$, where i is the number of a task, j is the number of a true action for a given task. A composition that is the consistent implementation of true actions, is a true action: $a_j^i * a_r^s, i, s = 1, \dots n, j, r = 1,,, l_i$. If an atomic true action a_j^i is made then the reverse action is a true action which is designated by $(a_j^i)^{-1}$ or by $a_j^{i^{-1}}$ and is named a reverse true action. Evidently $((a_j^i)^{-1})^{-1} = e$, where e is a neutral element. Evidently $((a_j^i)^{-1})^{-1} = e$. Let composition of true action be a true action. Then the full system of true actions of a given task can be described by a group $\wp = \langle T,*,^{-1}, e \rangle$. Then $\wp$ is a free group of finite rank $r = n\sum_{i=1}^{n} l_i$ with $\{a_j^i | i = 1, \dots n, j = 1,,, l_i\}$ as generates.

Theorem of true actions' description. The set of all true actions is defined by no more than two combinations of words which are finite length's compositions of atomic true actions.

Now consider the analysis of solutions offered by the pupil.

The pupil gives the full algorithm of decision of a task number *i*. After that expert writes down all compositions of atomic actions of the pupil: $\{\nu_\gamma\ (u_i^1, \dots, u_i^{h_i}) \mid i = 1, \dots, n, s = 1, \dots, h_i\}$, and examines a group Ꝁ, generated by the set of all atomic actions made by the pupil:

К=$\langle \{v_\gamma (u_i^1, \ldots, u_i^{h_i}) \mid i = 1, \ldots, n, s = 1, \ldots, h_i\}\rangle$ - the group of a decisions suggested by the pupil. Van Kampen's lemma allowed to see the group of a decisions of a task and to analyze it. Besides it one can construct the lattice of all subgroups of this group giving an opportunity to portray all possible moves the pupils in solving the problem. This group may be countable, not necessarily finite. The decision given by the pupil is true if К $\subset \wp$; К$\cap$ $\mathfrak{G} = \langle e \rangle$; and true answer is found out.

Let's concern Change-over to writing down in the first order language.

Writing down the true decisions and error decisions can be done in the first order language with the signature $\Omega = \langle *, \quad ^{-1}, e\rangle$ when one examines the elementary theory of the group of true decisions of a task and the elementary theory of the group of errors of a task.

Using proposed in the article approach to the formalization of the General Theory of Systems with the help of the theory of algebraic systems are obtained as applications substantial results in various fields of activity, such as the modeling of economic phenomena and processes, expert systems in General theory of teaching.

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